

Bending dynamics of bidomain LiNbO_3 single-crystalline bimorphs

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Abstract

This study examines the bending dynamics of cantilevered single-crystalline bimorphs made from bidomain LiNbO_3 (BLN), emphasizing the interaction between bending resonances, antiresonances, and torsional vibrations. Combining theoretical modeling and experimental validation, we demonstrate that a 1D analytical model, based on classical beam theory, surpasses FEM modeling in predicting bending resonances, particularly in the low-frequency range. The analytical model provides precise positioning of resonant frequencies, enabling the strategic alignment of torsional resonances with bending antiresonances, as confirmed experimentally. Our findings underscore the high predictability of BLN-based bimorphs, allowing accurate descriptions of their behavior at bending resonances, antiresonances, and piezoelectrically passive torsional resonances. Unlike glued actuators made of lead-based ceramics, BLN bimorphs offer superior thermal stability, long-term reliability, and predictable performance. These properties, combined with the availability of commercially produced LN crystals, position BLN-based bimorphs as ideal candidates for MEMS applications, enabling advanced sensing and actuation solutions with reduced reliance on complex feedback loops.

Keywords

piezoelectric bimorph, lithium niobate (LiNbO_3), one-dimensional model, bidomain crystal, single-crystal bimorph, precise positioning

1. Introduction

High predictability, resolution, and reproducibility, along with a wide dynamic range, are essential for precision positioning in modern scientific and technological equipment relying on mechanical movement, such as scanning probe microscopes, quantum resonators, photolithography tools, and adaptive optics. Piezoelectric actuators play a crucial role in the precision positioning systems of such equipment. Although current technology allows for large displacements under heavy loads with high

precision, there is an inherent trade-off: as the load and displacement increase, precision and reproducibility often decrease. This is because the strain produced by the piezoelectric effect in piezoelectric materials at moderate voltages is relatively low, typically resulting in displacements of only tens or hundreds of nanometers at voltages in the hundreds of volts. The maximum achievable load depends on the stiffness of the actuator's construction, as the force generated by the piezoelectric effect must move the target load rather than deform the material itself. Mechanical engineers have proposed various designs to

overcome these limitations, but as with a lever in the classical sense, one can enhance force at the cost of displacement or, conversely, increase maximum displacement at the expense of allowable load. Additionally, ratchet-like piezoelectric mechanisms can achieve high loads and displacements, but their predictability and resolution are often significantly lower than those of single-step actuators. Therefore, the effectiveness of piezoelectric-based positioning systems relies on two key components: the piezoelectric material and the design that translates electrical signals into mechanical displacement.

The most commonly used materials for piezoelectric actuation are lead zirconate titanate (PbZr_xTi_{1-x}O₃, PZT) ceramics with varying compositions, providing a wide range of piezoelectric and mechanical properties. The widespread use of PZT is attributed to its capability for producing piezoelectric transducers in various shapes and sizes through traditional ceramic processing methods, its high piezoelectric coefficients (in the hundreds of pC/N), and its global affordability and availability. Despite these advantages, actuators based on PZT ceramics are seldom used for precise actuation without additional feedback loops due to mechanical hysteresis, an intrinsic property of their polycrystalline nature caused by various interfering effects [1]. This hysteretic, non-linear piezoelectric behavior is observed not only in the ceramic state but also in single-crystal form [2], and their intrinsic properties are particularly sensitive to temperature variations [3] and even to changes in the amplitude of the driving voltage [4]. Therefore, even if an actuator is designed for linear, hysteresis-free operation, using PZT without additional compensation methods can undermine these improvements, resulting in displacements with poor reproducibility. Thus, the engineering of highly precise and reliable actuators must begin with the careful selection of appropriate material.

Ferroelectrics with a high Curie point (and classical piezoelectrics that are not ferroelectric but have a high melting point) in a single-crystalline state do not exhibit the disadvantages inherent to the PZT family, but their limited use in commercial actuators is primarily due to their lower piezoelectric coefficients compared to PZT. For example, the highest piezoelectric coefficients found in the matrix of piezoelectric modules (including values derived from other components, as detailed in [5]) are approximately 5 pC/N for quartz [6], around 10 pC/N for langatate [7], and approximately 8 pC/N for paratellurite [8]. However, as it was noted above, the lack in piezoelectrically induced deformation can be overcome by an appropriate actuator design.

Among various types of piezoelectric transducers with intricate shapes and internal structures, a straightforward yet highly effective approach to increasing maximum displacement – albeit at the expense of load capacity – is the adoption of “morph” designs, most commonly unimorphs and bimorphs. These designs are typically thin composite structures composed of two bonded layers. In a unimorph, one layer is piezoelectric with electrodes,

while the other is a non-piezoelectric (passive) layer that acts as a spring. In contrast, a bimorph has both layers as piezoelectric. These designs utilize transverse piezoelectricity, where applying voltage to the piezoelectric layer causes it to contract or expand along the length of the composite plate. When a driving voltage is applied, both designs bend: in a unimorph, the passive layer restricts movement, causing the actuator to bend, while in a bimorph, the second piezoelectric layer deforms in the opposite direction to the first, also resulting in bending but with greater force. The displacement perpendicular to the laminate surface due to the “morph effect” is significantly higher than that achieved along the laminate through transverse piezoelectricity alone. Morph laminates are generally secured within devices by clamping them at one end, both ends, or along their edges, as in membrane applications, with the largest displacements occurring at points farthest from the clamping locations. However, even when linear, non-hysteretic piezoelectric materials are used for bimorphs, challenges remain due to the epoxy bonding layer, which has different mechanical properties and thermal stability than the working layers. Consequently, issues such as hysteresis, creep, and thermal instability – though less pronounced than in PZT ceramics – still persist.

From this perspective, single-crystalline ferroelectric lithium niobate (LiNbO₃, LN) is a promising material for producing ultra-precise positioning actuators. A key advantage of LN over other single-crystalline piezoelectrics, such as quartz or langasite, is that LN-based piezoelectric transducers can be constructed without adhesives or high-temperature bonding; instead, the transducer's internal structure can be designed through domain engineering, owing to the ferroelectric nature of LN. Furthermore, the material's high coercive electric field enables the creation of stable domain configurations that can withstand high operational voltages, while the uniaxial domain structure simplifies theoretical modeling of the transducers' behavior. LN exhibits piezoelectric coefficients in the range of tens of pC/N [9], an order of magnitude higher than most classical single-crystalline piezoelectrics and one to two orders of magnitude lower than lead-based ferroelectric relaxors. LN provides a fast, hysteresis-free piezoelectric response to an externally applied voltage [10], along with a high Curie point and minimal variation in piezoelectric properties across a broad temperature range [11] making it superior to PZT in these respects. As a result, LN has been the subject of extensive research for applications in high-quality, lead-free piezoelectric sensors, vibrational energy harvesters [12–14], and, to a lesser extent, actuators [15–18].

A LN-based single-crystalline bimorph can be fabricated from an LN crystal plate using various methods of domain engineering [19]. The simplest approach involves annealing an initially single-domain crystalline plate near LN's Curie point, inducing the formation of a thin layer with an inverted spontaneous polarization vector on the originally positive face [20]. This process creates

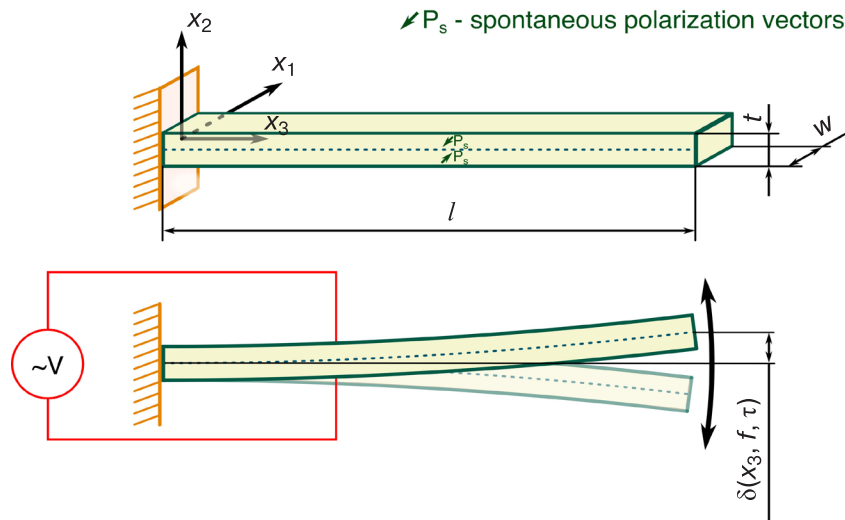


Figure 1. Schematic representation of the coordinate system defined for the bimorph and its bending deformation under voltage excitation

a ferroelectric domain structure with a head-to-head configuration and a domain wall running parallel to the crystal plate's surface. The distance between the domain wall and the surface can be controlled by adjusting the annealing temperature and duration. This structure is commonly referred to as “bidomain” lithium niobate (BLN). In BLN, the only boundary between the oppositely polarized ferroelectric layers is the ferroelectric domain wall, eliminating the need for adhesives or intergrain boundaries. The oppositely polarized domains can act as active layers of a bimorph when voltage is applied to the polar faces. Since the transverse piezoelectric coefficient is low in the z -cut of LN, the optimal (among commercially available [21]) cut for a bimorph actuator is the $y+128^\circ$ -cut, where the working coefficient $d_{23} \approx 26$ pC/N [22]. Recent studies have demonstrated the high potential of BLN for use in sensors [23–26], actuators [27–30], energy harvesters [31], and electron emitters [32]. In our previous work [33], we experimentally validated a simple 1D model of an ideal bimorph, as proposed by Smits et al. [34], for a BLN-based bimorph bender, showing strong agreement between the model's predictions and the experimentally measured displacements and impedance values in the frequency range that included the first three bending resonances and antiresonances of the investigated sample. This close alignment supports the development of a new generation of precise positioning systems with highly predictable behavior. Additionally, we demonstrated the capability to measure the transverse piezoelectric coefficient, longitudinal mechanical compliance, dielectric permittivity, and piezoelectric coupling coefficient of the $y+128^\circ$ -cut LN crystal using the actuator as a specimen, by solving the inverse problem for displacement and impedance data through simulated annealing optimization.

The goal of this paper is to uncover key patterns in the bending behavior of BLN-based single-crystal bimorphs. We examine the shapes of the bimorph at various excitation frequencies and analyze critical positions along its length by comparing our experimental results with

analytical modeling based on the 1D theory proposed by Smits et al. [34], as well as finite element method (FEM) simulations of the deformed bidomain crystal shape.

2. One-dimensional theory of bimorph bending

The frequency-dependent properties of piezoelectric unimorphs and bimorphs have been modeled for the one-dimensional case in works [34–38], and here we briefly summarize the most important points.

In this study, we consider a symmetrical piezoelectric bimorph in the form of a thin rectangular plate, suitable for classical plate theory, with a width much smaller than its length and clamped at one end. The plate has dimensions of length (l), width (w), and thickness (t). A Cartesian coordinate system for the further calculations is defined with the x_1 -axis along the width of the bimorph, x_2 -axis along the thickness, and the x_3 -axis along the length, as illustrated in Fig. 1.

In the subsequent analysis, we assume that the bimorph actuator is fabricated from a rectangular plate of $y+128^\circ$ -cut BLN crystal with a congruent composition. The long side of the plate is oriented perpendicular to the non-polar x_1 crystallophysical axis, while the x_2 -axis, normal to the plate, aligns with the x'_2 -axis obtained by counterclockwise rotation of the crystallophysical Cartesian coordinate system around the x_1 -axis by 128° .

For stationary oscillations of an unloaded piezoelectric bimorph without an intermediate elastic layer, excited by a harmonic voltage signal with frequency f and amplitude V , the displacement of any point along the x_3 axis at time τ can be calculated as $\delta(x_3, f, \tau, V) = \delta_{\text{amp}}(x_3, f) \times V \cos(2\pi f \tau)$. Since the amplitude of displacement is the experimentally measurable characteristic when the harmonic voltage excitation is applied to the bimorph, the voltage-normalized amplitude $\delta_{\text{amp}}(x_3, f)$ is of primary

interest. According to the theory by Smits et al. [34], $\delta_{\text{amp}}(x_3, f)$ can be defined:

$$\delta_{\text{amp}}(x_3, f) = \frac{3d_{23}Y(\Omega, x_3)}{2t^2\Omega^2[1 + \cos(\Omega l)\cosh(\Omega l)]}, \quad (1)$$

$$Y(\Omega, x_3) = [-\cos(\Omega l) - \cosh(\Omega l)] \times \\ \times [\cos(\Omega x_3) - \cosh(\Omega x_3)] + [-\sin(\Omega l) + \\ + \sinh(\Omega l)][\sin(\Omega x_3) - \sinh(\Omega x_3)], \quad (2)$$

where $\Omega = 2\sqrt{\frac{\pi f}{t}}\sqrt{3\rho s_{33}^E}$; ρ – material's density measured at constant temperature; s_{33}^E – mechanical compliance in the direction of x_3 axis, measured in electrically free state at constant temperature; d_{23} – transverse piezoelectric coefficient measured at constant temperature.

Equation (1) enables modeling the shapes of the bimorph at any given moment and allows for the prediction of the natural frequencies of mechanical oscillations. In the limiting case of $f \rightarrow 0$ Eq. (1) simplifies to the formula for the static displacement of a symmetrical homogeneous bimorph under a constant applied voltage, as described in [37]:

$$\delta_{\text{amp}}(l, 0) = \frac{3}{2}d_{23}\left(\frac{l}{t}\right)^2. \quad (3)$$

When the denominator multiplier $[1 + \cos(\Omega l)\cosh(\Omega l)]$ approaches zero, the solution diverges, and the value of δ becomes undefined. Consequently, the classical well-known condition for resonant frequencies can be expressed as the set of solutions of the transcendental equation:

$$\cos(\Omega_{r,n}l) \cdot \cosh(\Omega_{r,n}l) = -1, \quad n = 1, 2, 3, \dots \quad (4)$$

or using hyperbolic secant function:

$$\cos(\Omega_{r,n}l) = -\text{sech}(\Omega_{r,n}l), \quad n = 1, 2, 3, \dots \quad (4^*)$$

From this, the formulation providing approximate values for the natural frequencies of the first few bending modes is:

$$f_{r,n} = \frac{\Omega_{r,n}^2 t}{4\pi\sqrt{3\rho s_{33}^E}} = \frac{t}{l^2\sqrt{3\rho s_{33}^E}} \frac{Q_n^2}{4\pi}, \quad n = 1, 2, 3, \dots \quad (4^{**})$$

where $Q_n = \Omega_{r,n}l$, and the approximate values for the first five resonant frequencies are given by: $Q_1 \approx 1.8751$, $Q_2 \approx 4.6941$, $Q_3 \approx 7.8548$, $Q_4 \approx 10.9955$, $Q_5 \approx 14.1372$, ...

At the free end of the bimorph $x_3 = l$, the multiplier $Y(\Omega, x_3)$ given by Eq. (2) is simplified to $Y(\Omega, l) = 2\sin(\Omega l) \cdot \sinh(\Omega l)$. This indicates that there exist frequencies at which $Y(\Omega_{a,n}, l) = 0$ which corresponds to the condition

$$\sin(\Omega_{a,n}l) = 0, \quad n = 1, 2, 3, \dots \quad (5)$$

The solutions of this equation are $\Omega_{a,n}l = \pi n$ with $n = 1, 2, 3, \dots$. In this case, there is no displacement of the free end along the x_2 -axis at any moment during the sinusoidal voltage excitation of the bimorph, even though all other points along the bimorph are displaced along the x_2 -axis. These frequencies are characterized as bending antiresonances with the frequencies of:

$$f_{a,n} = \frac{\Omega_{a,n}^2 t}{4\pi\sqrt{3\rho s_{33}^E}} = \frac{t}{l^2\sqrt{3\rho s_{33}^E}} \frac{\pi n^2}{4}, \quad n = 1, 2, 3, \dots \quad (5^*)$$

This situation was precisely described in the paper by Ballato and Smits [38], where it was noted that one encounters a “mixed situation: resonances determined by the ‘typical beam’ behavior with its implicit solutions, and the antiresonances given by the ‘typical string’ behavior with zeros at multiples of π ”. Interestingly, bending antiresonant frequencies – i.e., frequencies of external harmonic excitation at which there is no displacement of the free end, as observed in piezoelectric beams – are not typically predicted in non-piezoelectric cantilever beams under standard periodic bending deformation [39, 40] or when excited by vibrations at the clamped end [41, 42], at least in the absence of attachments or complex constraints applied to the free end of the beam [43]. We should also note that, while the resonant solutions of Eq. (4) correspond to the bending beam as a whole, the antiresonant solutions of Eq. (5) correspond only to the free end of the beam ($x_3 = l$ position). Strictly speaking, the frequencies at which another point ($x_3 \neq l$) of the beam will not move and exhibit “antiresonant” behavior will differ from the solution of Eq. (5) and can be found by solving the equation $Y(\Omega, x_3) = 0$.

Since the function $[-\text{sech}(\Omega_{r,n}l)]$ from Eq. (4*) asymptotically approaches zero, starting from the third solution ($n = 3, 4, 5, \dots$), the values of $Q_n \approx [(2n - 1)\pi]/2$ with good accuracy. The positions of the higher-order resonances and antiresonances can thus be described in terms of Ωl as the roots of the equations $\cos(\Omega_{r,n}l) = 0$ for resonances and $\sin(\Omega_{a,n}l) = 0$ for antiresonances. Fig. 2 graphically illustrates the positions of the first several resonances and antiresonances of a rectangular piezoelectric bimorph cantilever in terms of the Ωl argument.

Since both resonance and antiresonance phenomena are determined by the periodicity of the $\cos(\Omega l)$ and $\sin(\Omega l)$ functions, the sign of δ_{amp} changes each time the frequency of the bimorph's voltage excitation crosses a resonance or antiresonance frequency. This results in a phase shift of π between the excitation signal and the displacement response at the free end of the bimorph. Although no phase shift is predicted before the first bending resonance, the sign of the piezoelectric coefficient and the ability to fabricate bimorphs in two types (with polarization vectors in the layers directed either toward or away from each other) can introduce an initial phase shift of π relative to the excitation signal due to the physical installation. Therefore, depending on the orientation, the free end of the bimorph may move either toward or

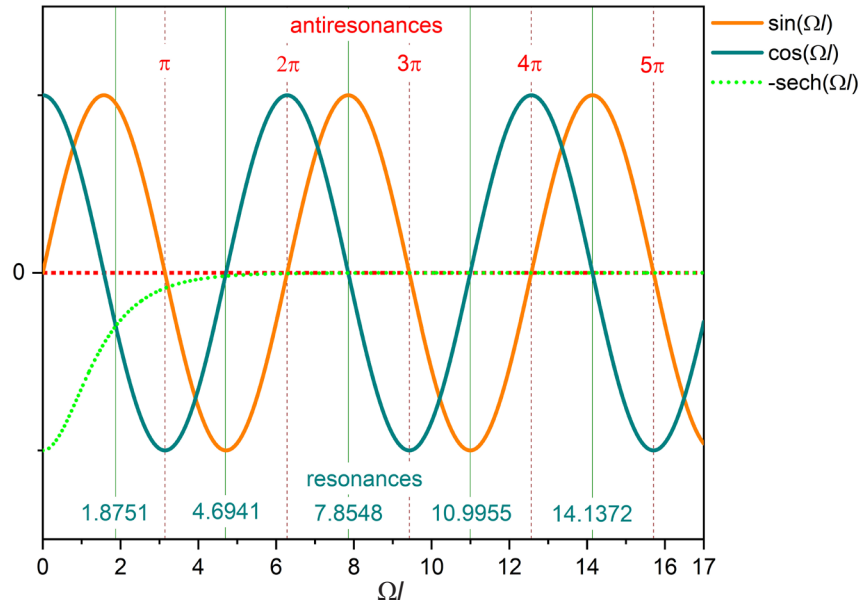


Figure 2. Positions of bimorph resonances and antiresonances in terms of Ω/l . Bending resonances are determined by the intersections of the dark-green and light-green lines, corresponding to Eq. (4**), while antiresonances are determined by the intersections of the orange and red lines, corresponding to Eq. (5*). Note that while the positions of antiresonance frequencies are always determined by the condition $\sin(\Omega_{a,n}l) = 0$, the accurate positions of resonant frequencies are determined by the transcendental Eq. (4**). However, starting from the third solution of (4**) (for $n = 3, 4, 5, \dots$), these positions can be approximated by the condition $\cos(\Omega_{r,n}l) = 0$

away from the observer when a positive voltage is applied.

It should be noted that model (1) does not account for mechanical damping, such as viscous air losses. Consequently, unlike oscillating systems with dissipation, resonant frequencies f_r cause divergence in the model. As a result, the bent bimorph's shape and displacement amplitude along its length cannot be modeled precisely at the resonant frequency using Eq. (1), and exact displacement values near resonance cannot be calculated. However, because resonant peaks are sharp and narrow in the damping-free approach, a slight shift in the excitation frequency to either side of the resonance allows

for modeling the bimorph's shape in its vicinity. Another consequence of the divergence in Eq. (1) at the resonance frequency is that displacements near different resonances become difficult to compare, meaning that the true displacement magnitudes can only be determined experimentally. Schematic representations of the normalized vibrating bimorph shapes (amplitude values) at various frequencies, including those near resonances, as predicted by Eq. (1), are shown in Fig. 3.

Generally, the effect of viscous damping is to shift the undamped resonance peak f_r to lower frequencies and smooth the π -radian phase jump in the phase response $\theta(f)$ (the phase difference between the excitation signal

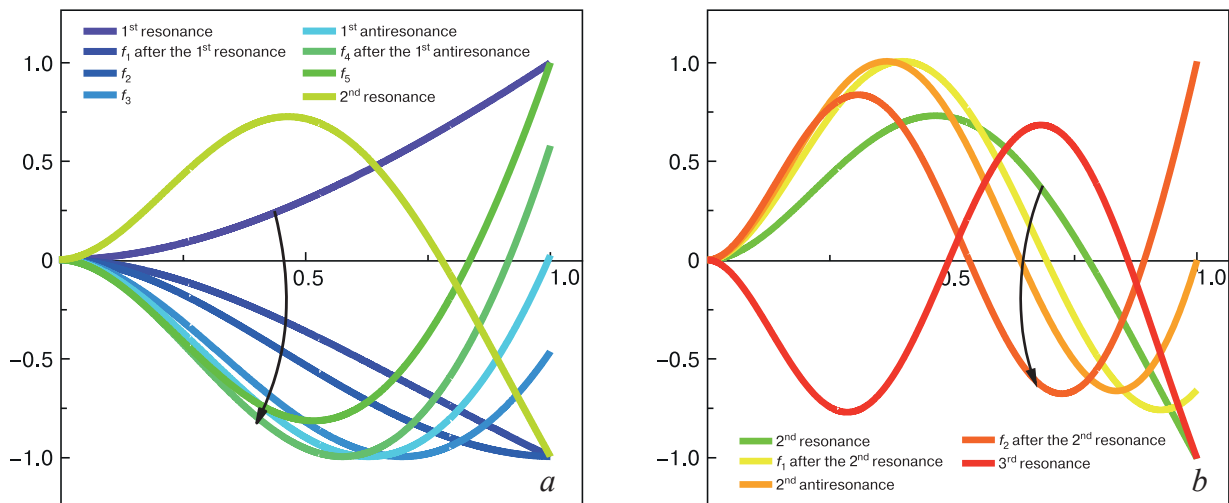


Figure 3. Representations of the bimorph's shape at selected frequencies (normalized) predicted by Eq. (1). Arrows indicate increasing frequency: (a) between the 1st and 2nd bending resonances, (b) between the 2nd and 3rd bending resonances

and displacement). For low-frequency vibrations, damping from the surrounding air is minimal. This is evidenced by sharp resonance peaks in experimental data for BLN-based bimorphs [33], which can be partially attributed to the low internal losses of the single-crystal, adhesive-less beam and partially to the actuator's low speed near several first resonances (due to both low frequencies and small displacements). Since the crystal remains intact and does not degrade even under moderate voltage excitations close to the resonance frequency, it can be inferred that damping does emerge in such cases, and nonlinear terms in the $\delta(V)$ dependence become significant.

In the vast majority of practical applications of bimorph actuators, low-damped excitations at natural beam frequencies are avoided. Nevertheless, it remains possible to model the shape of the bimorph at frequencies near bending resonances using Eq. (1), without the need to explicitly include damping, at least within the low-frequency range. However, the upper limit of this “undamped” frequency range requires further definition.

An important point to note is that one-dimensional model (1) provides information on a bimorph's bending resonances and antiresonances, specifically those with displacements along the x_2 -axis.

However, if the bimorph's width is not significantly larger than its thickness (no more than a few times), lateral bending resonances, with displacements of the free end along the x_1 -axis, can also occur. A zeroth-order approximation of their frequencies can be obtained using Eq. (4**), substituting the bimorph's width w in place of the thickness t .

Additionally, in the low-frequency range where bending resonances occur, any long, narrow, and thin cantilevered plate – whether piezoelectric or not – also exhibits at least one non-bending resonance, specifically the torsion mode. This mode manifests as periodic twisting of the beam along its length. The frequency of the first torsional natural mode for a rectangular plate of length l , as considered here, can be accurately predicted [44] using the following formula:

$$f_{1,t} = \frac{1}{4l} \sqrt{\frac{GK}{\rho I_p}}, \quad (6)$$

where K is the torsion constant of the beam; I_p is the polar moment of inertia of the beam; G is the shear modulus.

For a thin plate with a rectangular cross-section of width w and thickness t , the torsion constant and polar moment of inertia can be calculated with sufficient accuracy as:

$$K = \frac{wt^3}{3}, \quad I_p = \frac{w^3t}{12}.$$

The shear modulus for an anisotropic material is direction-dependent. In the case of a cross-section perpendicular to the x_1x_2 plane, it is given by:

$$G = \frac{1}{2(s_{11}^E - s_{12}^E)},$$

where s_{11}^E and s_{12}^E are elements of the mechanical compliance matrix, measured in the electrically free state at constant temperature and calculated for the $y+128$ BLN crystal cut. Substituting these values into Eq. (6), the expression simplifies to:

$$f_{1,t} = \frac{t}{lw\sqrt{8\rho(s_{11}^E - s_{12}^E)}}. \quad (6^*)$$

Although lateral bending resonances and torsional resonances are inherent to non-piezoelectric beams, an important question is whether these modes can be directly excited by an externally applied voltage to the working electrodes of the bimorph (those covering the x_1x_3 -plane of the actuator in Fig. 1). If the resonant vibration of a specific deformation mode can be excited through the piezoelectric effect, we can also expect the presence of antiresonant frequencies for that mode. Conversely, if the vibration is indirectly excited as a result of the primary (bending) deformations of the bimorph in the vicinity of the specific natural frequency, no corresponding antiresonances will be observed.

For the bimorph design, it is evident that piezoelectrically driven bending is inherently a result of the laminate structure, with the normal to the layers directed along the axis of bending. Consequently, pure lateral bending in the bimorph is restricted.

To address the question of whether piezoelectrically driven torsional vibrations can occur, it is necessary to analyze the piezoelectric matrix of the material used to fabricate the transducer. For the twisting of the beam around the long symmetry axis, the top and bottom layers must exhibit shear deformations of opposite signs in the x_1x_3 -plane when the electric field vector is directed along x_2 -axis (across the thickness of the bimorph). This means that, for piezoelectrically driven torsional deformation to occur, the crystal must possess a non-zero piezoelectric coefficient d_{25} .

In the case of BLN crystal, with piezoelectric coefficients in the standard crystal orientation of $3m$ point group given as $d_{15} = 67.7$ pC/N, $d_{22} = 18.9$ pC/N, $d_{31} = -1.0$ pC/N, $d_{33} = 7.9$ pC/N [11], the piezoelectric matrix for the $y+128^\circ$ -cut orientation – where the long side of the rectangular plate is oriented perpendicular to the non-polar x_1 crystallophysical axis – can be calculated as:

$$\mathbf{d}^{y+128^\circ} = \begin{pmatrix} 0 & 0 & 0 & 0 & -11.89 & 76.62 \\ 10.85 & 19.38 & -25.57 & -8.01 & 0 & 0 \\ 15.51 & 17.45 & -36.59 & 3.77 & 0 & 0 \end{pmatrix} \text{pC/N}$$

It is evident that, for a crystal of point group $3m$, a single rotation of the standard Cartesian crystallophysical coordinate system around the x_1 -axis does not result in the appearance of a non-zero d_{25} piezoelectric coefficient. Therefore, in the considered configuration, the bimorph does not exhibit twisting around the x_3 -axis when an electric field is

applied along the x_2 -axis, and no torsional antiresonances are observed. It is worth noting, however, that if the crystal plate for a bimorph actuator is fabricated from the same $y+128^\circ$ -cut wafer but oriented such that the long side of the BLN crystal is neither parallel nor perpendicular to the non-polar x_1 -axis, then the d_{25} coefficient becomes non-zero. In this case, piezoelectrically driven torsional vibrations are possible. Furthermore, by selecting an appropriate orientation angle, it may even be possible to suppress piezoelectrically driven bending [45].

Since lateral bending and torsional vibrations in the considered BLN-based actuator are only spontaneously excited by external impacts – such as piezoelectrically driven bending vibrations – the question arises whether these vibrations can be detected experimentally. While lateral bending exhibits displacements primarily along the x_1 -axis, these are expected to have a small magnitude due to the high rigidity of the actuator, making their detection a very challenging task. In contrast, torsional vibrations are characterized by pronounced shifts of the edges across the width of the bimorph in the vicinity of the torsional resonance frequency. These displacements, occurring along the x_2 -axis, can be detected using laser interferometry.

It is expected that the magnitude of these torsional displacements will be orders of magnitude smaller than the bending displacements under external voltage excitation, making them difficult to detect in general. However, the BLN-based bimorph can be designed such that the torsional resonance frequency coincides with one of the bending antiresonance frequencies. In this scenario, the twisting motion could be readily observed near the free end of the beam, where there are no significant bending displacements in the background.

In this study we decided to choose the dimensions of the actuator such that the 2nd bending antiresonance matched with the 1st torsional resonance. Equating formulas (5*) and (6*) one obtains the following condition for the BLN-based bimorph size:

$$\frac{l}{w} \approx 5.13 \sqrt{\frac{(s_{11}^E - s_{12}^E)}{s_{33}^E}}.$$

By calculating the matrix of mechanical compliances for the BLN crystal plate under consideration with the $y+128^\circ$ -cut orientation, and using the material constants published in [46], we obtain the following result in a manner similar to the calculation of the piezoelectric coefficients:

$$\mathbf{s}^E, y+128^\circ = \begin{pmatrix} 5.83 & -0.85 & -1.75 & 0.53 & 0 & 0 \\ -0.85 & 5.75 & -2.36 & 0.22 & 0 & 0 \\ -1.75 & -2.36 & 6.91 & 0.32 & 0 & 0 \\ 0.53 & 0.22 & 0.32 & 13.49 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13.21 & -1.04 \\ 0 & 0 & 0 & 0 & -1.04 & 17.85 \end{pmatrix} \text{TPa}^{-1}$$

Thus, to satisfy the condition $f_{a,2} = f_{t,1}$, the BLN-based bimorph should be fabricated with a length-to-width ratio of approximately $l/w \approx 5.05$.

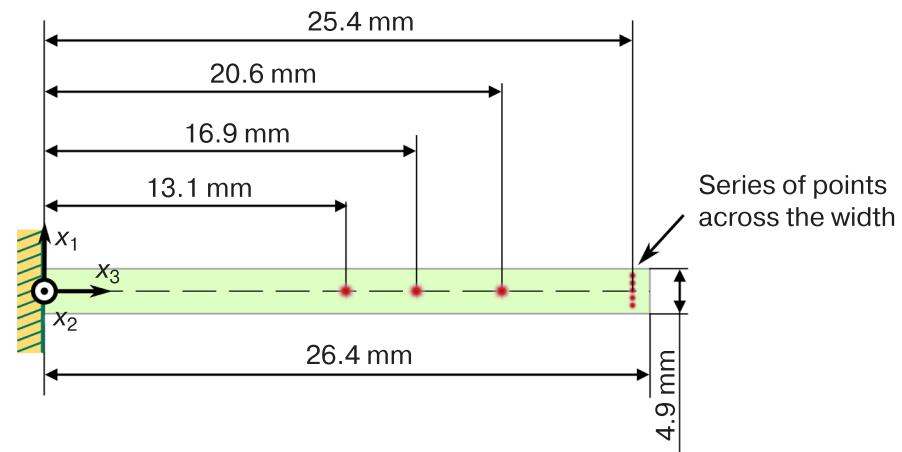
3. Experimental

For further experimental analysis, we produced a rectangular plate bimorph actuator from $y+128^\circ$ -cut BLN crystal with a congruent composition. The bidomain ferroelectric configuration was formed in the LN sample through Li_2O out-diffusion annealing [19]. Thin titanium film electrodes, approximately 200 nm thick, were applied to the working surfaces of the BLN crystal via magnetron sputtering to form the bimorph actuator. The BLN crystal was then secured in a custom-made brass clamp and mounted onto a precision 2D translation stage provided by ThorLabs. The final cantilevered bimorph had dimensions of $l \times w \times t = 26.4 \times 4.9 \times 0.515 \text{ mm}^3$. Length and width were measured with a caliper with a precision of 0.1 mm, while the thickness was measured using a micrometer screw gauge with an accuracy of 0.005 mm. The initial target for the free length was 25 mm, designed to satisfy the previously introduced condition $l/w \approx 5.05$, but due to the challenges of securely fastening the crystal while maintaining precise geometric control, the actual obtained value of 26.4 mm, resulting in a slightly higher ratio $l/w \approx 5.39$, was deemed satisfactory.

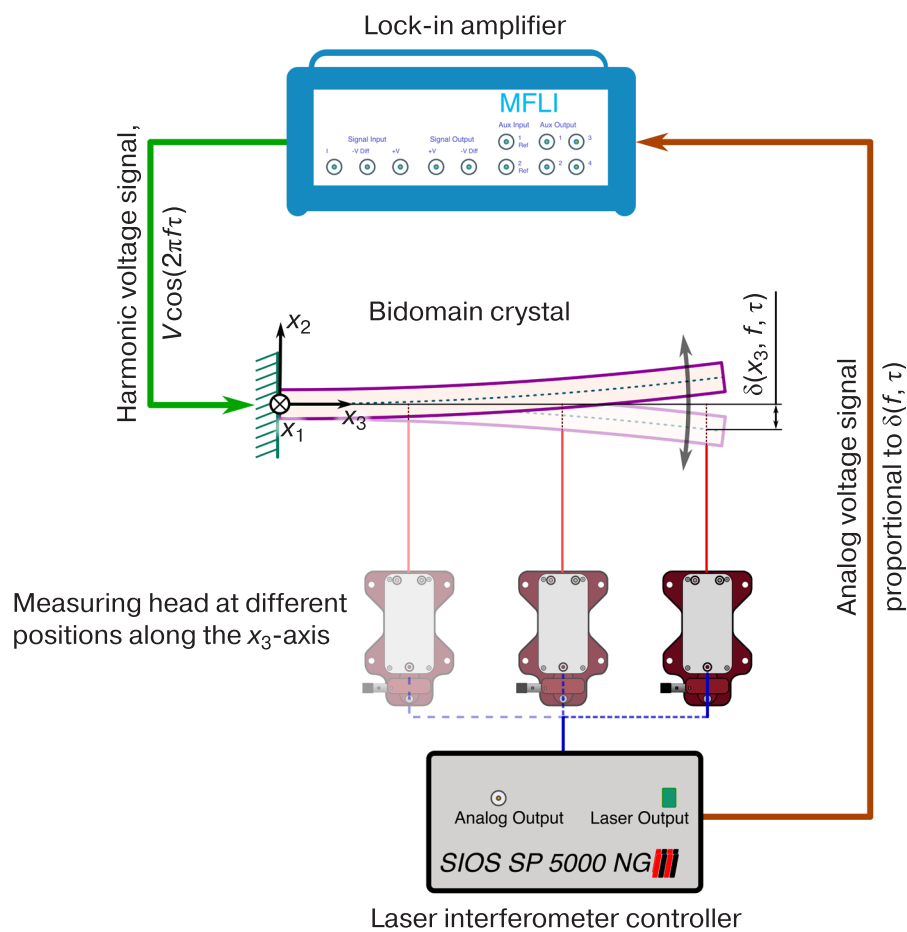
The experimental dependencies of the displacement amplitude, denoted as $\delta(x_3, x_1, f)$, on frequency f , longitudinal position x_3 and position along width of the bimorph x_1 , were measured using an SP 5000 NG laser interferometer equipped with the SM-05 Vibration-Analysis Module providing an analog output (SIOS Messtechnik). The measuring laser spot was focused on the sample using a lens with a focal distance of 50 mm. Displacements were measured at two sets of spatial positions, with shifts between measurements achieved by translating the sample on a 2D precision stage.

First, the dependence of displacement on the longitudinal coordinate x_3 along the length of the bimorph was measured at points located along the center of the actuator's width (denoted as $x_1 = 0$), with the resulting data expressed as $\delta(x_3, f) = \delta(x_3, x_1 = 0, f)$ below. The specific x_3 positions for these measurements $\delta(x_3, f)$ were $x_3 = 25.4 \text{ mm}$ (1 mm from the free end of the bimorph), $x_3 = 20.6 \text{ mm}$ (the vicinity of the 2nd bending resonant node), $x_3 = 16.9 \text{ mm}$ (the vicinity of the 2nd bending antiresonant node), and $x_3 = 13.1 \text{ mm}$ (the vicinity of the 3rd bending resonant node).

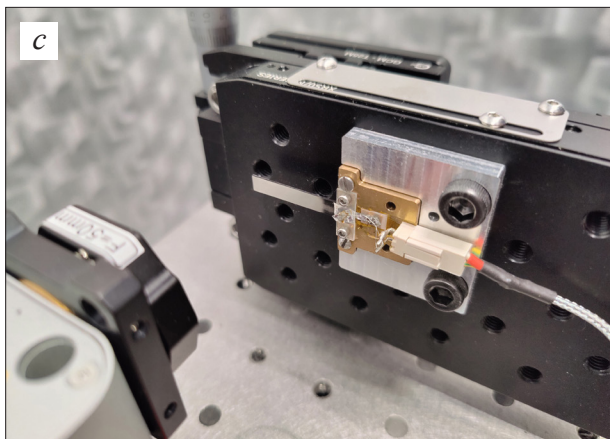
Separately, near the second bending antiresonance (frequency range between 7.8 kHz and 8.8 kHz), the dependence of displacement on the transverse coordinate x_1 across the width of the bimorph was measured. Data were collected at points 1 mm before the free end of the bimorph bender ($x_3 = 25.4 \text{ mm}$), with x_1 coordinates incremented in 0.5 mm steps between the two extreme positions: 0.5 mm from the left edge and 0.5 mm from the



a



b



c

Figure 4. (a) Spatial positions for displacement measurements, (b) schematic illustration of the measurement setup, (c) photograph of the BLN-based bimorph mounted on a precision positioning stage. Displacement shifts between measurement points were achieved by moving the sample on a 2D precision stage

right edge of the cantilever. These datasets are denoted below as $\delta_{l-1}(x_1, f) = \delta(x_3 = l - 1, x_1, f)$.

The spatial positions of the points for measuring displacements are shown schematically in Fig. 4a.

For displacement measurements, a sinusoidal voltage excitation signal $V\cos(2\pi ft)$ with voltage amplitude V , and frequency f , was applied to the electrodes of the bimorph actuator using a signal generator from an MFLI lock-in amplifier (Zurich Instruments). To measure the amplitude of the periodic vibrations of the BLN-based bimorph, the analog voltage signal from the interferometer, proportional to the measured displacements, was then fed into the input of the lock-in amplifier. Simultaneously, the root mean square (RMS) value and phase of the interferometer's output signal were recorded across the frequency range of 10 Hz to 25 kHz, with a 10 Hz increment and a lock-in low-pass filter bandwidth set to 1 Hz. In the vicinity of the second bending antiresonance (frequency range between 7.8 kHz and 8.8 kHz), displacements across the width of the bimorph were measured with a finer frequency increment of 1 Hz, using the same bandwidth. The reasons for this finer resolution are provided below. The measured data were subsequently converted into displacement magnitude values. An illustration of the measurement setup, as well as a photograph of the sample under investigation mounted on a precision positioning stage, are shown in Fig. 4 (b and c), respectively.

Measurements were conducted at room temperature (approximately 23 °C). No measures were taken to stabilize the ambient temperature, as slight temperature fluctuations had no observable effect on the measurement results.

Figure 5a shows the graph of the displacement magnitude measured near the free edge ($x_2 = l - 1$ mm = 25.4 mm) of the BLN-based bimorph under study. The low-frequency range contains two concatenated datasets measured separately. The first dataset, represented by dark-red triangles, was measured in the range of 10 Hz to 6 kHz with an excitation voltage magnitude of 0.15 V. The second dataset, represented by blue squares, was measured from 3 kHz to 25 kHz with an excitation voltage magnitude of 1 V. Both datasets were normalized to an excitation magnitude of 1 V.

Separating the two measurements allowed for an increased displacement response in the higher-frequency range, improved signal-to-noise ratio near antiresonances, and avoidance of saturation at the first bending resonance. As seen in Fig. 5a, the two datasets match closely, with a maximum discrepancy of 10 %, despite the more than sixfold difference in excitation amplitude. This demonstrates a linear dependence of the displacement response on the magnitude of the driving signal.

The remaining discrepancy is primarily attributed to the lower signal-to-noise ratio in the dataset collected in the lower-frequency range, leading to higher uncertainty at small displacements. Notably, at the edge of the lower-frequency dataset where the largest discrepancy occurs, the voltage sensitivity is approximately 4 nm/V, corresponding to a maximum displacement of 0.6 nm under an excitation of 0.15 V – a value that poses a significant challenge for accurate measurement.

The experimental data measured near the free end of the investigated BLN-based bimorph were approximated using the model described by Eq. (1), following the method outlined in [33] (solving the inverse problem using the

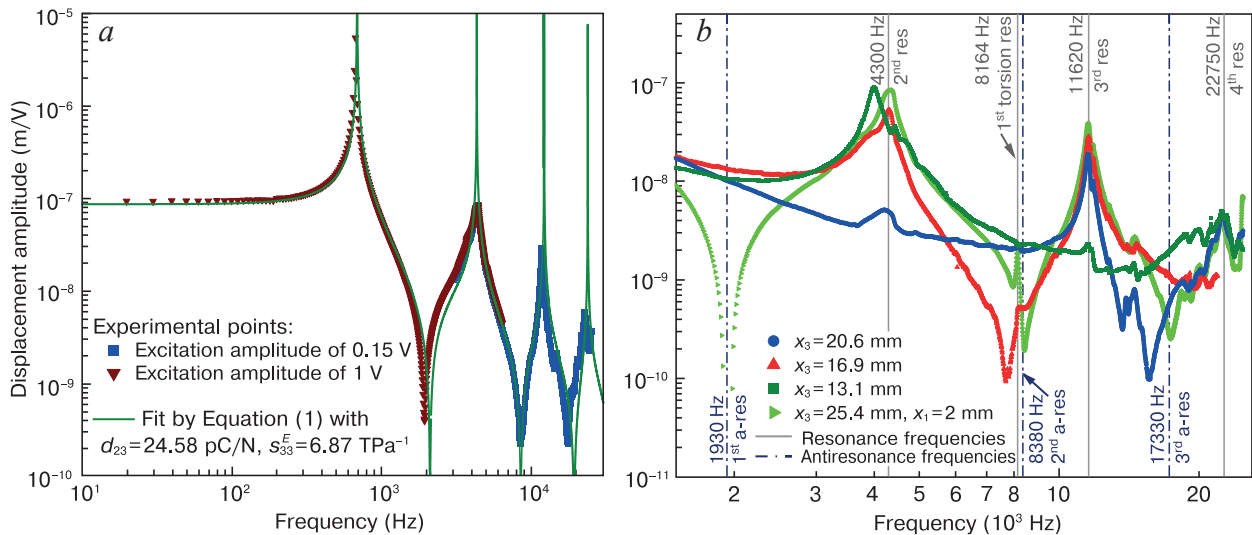


Figure 5. (a) Voltage-normalized displacement magnitude measured at the center of the width ($x_1 = 0$) near the free edge ($x_3 = l - 1$ mm = 25.4 mm) of the BLN crystal, shown alongside the fitted curve based on Eq. (1), (b) Voltage-normalized displacement magnitude measured at points with coordinates $x_3 = 20.6$ mm (the vicinity of the 2nd bending resonant node), $x_3 = 16.9$ mm (the vicinity of the 2nd bending antiresonant node), and 13.1 mm (the vicinity of the 3rd bending resonant node). Additionally, in (b), the displacement magnitude measured at one of the corners of the bimorph's free end ($x_3 = 25.4$ mm, $x_1 = 2$ mm) is included. The shorter length of the red curve at the right edge results from data for higher frequencies not being collected at this position, due to the noisy appearance of the low-level signal at $f > 22$ kHz

simulated annealing algorithm). The analysis was conducted on the fabricated $y+128^\circ$ -cut BLN crystal, with the axis orientations as defined above, and dimensions of length ($l = 26.4$ mm), thickness ($t = 0.515$ mm), and width ($w = 4.9$ mm). For the modeling, the density value $\rho = 4628$ kg/m³ was used, based on literature data for congruent composition [9]. The results obtained during the minimization of the least square residuals function are as follows: a transverse piezoelectric coefficient of $d_{23} = 24.58$ pC/N and mechanical compliance of $s_{33}^E = 6.87$ TPa⁻¹. These values are in good agreement with the matrices calculated for the $y+128^\circ$ -cut BLN crystal in Section 2. The results of the fitting are shown in Fig. 5a, alongside the experimental data points.

Figure 5b shows the graph of the displacement magnitude measured at three different points along the x_3 -axis, corresponding to nodes of displacement magnitude: $x_3 = 20.6$ mm (in the vicinity of the 2nd bending resonant node), $x_3 = 16.9$ mm (in the vicinity of the 2nd bending antiresonant node), and $x_3 = 13.1$ mm (in the vicinity of the 3rd bending resonant node). It can be observed that the positions of the second resonant peaks slightly differ between the points, while the third resonant peak occurs at the same frequency for all points. Despite this discrepancy, the overall shapes of the results are in good agreement with the predicted mode shapes (see Fig. 3 for comparison). The smallest displacements are observed at $x_3 = 16.9$ mm, which exhibits a strong minimum of displacement. However, the two other datasets also show a significant reduction in displacement at their respective characteristic frequencies.

Two important points should be noted:

1. The point at $x_3 = 20.6$ mm is not only in the vicinity of the 2nd bending resonant node but is also close to the node of the 3rd antiresonance (approximately 17330 Hz), which has an approximate x_3 -axis coordinate of 20.06 mm. This explains the minimum observed in the blue dataset in Fig. 5b.

2. Due to the complex nature of the vibrations and slight misalignment between the true node positions and those set by the micrometer screw gauge, the points at $x_3 = 20.6$ mm and $x_3 = 13.1$ mm do not exhibit deep minima but rather points with an absence of strong displacements. This contrasts with the case of the antiresonance at $x_3 = 16.9$ mm, where a deep minimum is clearly observed.

It is evident that, although the fitting curve shows good agreement with the experimental data, the best correspondence is observed near the first and third resonances, as well as the second antiresonant frequency. Several factors may contribute to this outcome:

1. Non-linearity of the regression problem. The regression problem solved during the fitting process exhibits strongly non-linear behavior, particularly near the resonant frequencies. At these points, both the measured data values and the absolute measurement errors are significantly larger than at other frequencies, primarily due to excitation by surrounding acoustic and vibrational noise. Consequently, the higher the value of a data point

in the dataset, the larger its residual, and therefore, the greater its contribution to the least-square goal function. In contrast, near antiresonances, displacement values are almost zero. However, the presence of acoustic noise, interference from non-zero displacements in neighboring points, and difficulties in precisely determining the spatial position along the beam lead to non-zero measured displacements with magnitudes of approximately 50 pm. Displacements of such small magnitudes are measured with high absolute error, making their contribution to the least-square goal function undesirable.

2. Non-ideality of the clamp. Imperfections in the clamp can result in resonance peak broadening, the appearance of side peaks, and mutual interference between modes. This effect is particularly evident in the vicinity of high-frequency resonant modes.

3. Neglect of damping in the model. The model by Smits does not account for damping effects, such as air resistance or internal material losses. Even the simplest case of viscous damping would shift experimental resonance peaks to lower frequencies, while the fitting is performed on experimental data assuming no damping. This discrepancy could also contribute to deviations between the model and experimental data. In our analysis, we prioritized points measured before the first resonance and between the first resonance and antiresonance, while largely neglecting points corresponding to very large or very small displacements, as these tend to introduce greater errors and less reliable contributions to the fitting process.

From this perspective, it is interesting to compare the results of modeling obtained using the 1D approach by Smits et al. (Eq. (1)) with those from the widely utilized FEM modeling approach. Based on the dimensions of the crystal under study, we modeled a BLN-based bimorph using FEM with a design consisting of two mechanically bonded layers of $y+128^\circ$ -cut LN crystals. The layers have opposite orientations of the x'_2 -axis, which is obtained by a counterclockwise rotation of 128° around the x_1 -axis from the standard crystallophysical x_2 -axis of the $3m$ point symmetry group (denoted as x_2 in Fig. 1 in previous discussion). The dimensions of the modeled crystal are as follows: free length ($l = 26.4$ mm), thickness ($t = 0.515$ mm, which is twice the thickness of each layer in the model), and width ($w = 4.9$ mm). The slight inconsistency between the required value of $l/w \approx 5.05$ and the actual value used in the modeling ($l/w \approx 5.39$) arises from the constraints of the real experimental setup geometry, where precise control of the crystal's free length during fastening is a challenging task. Nevertheless, as Eq. (6*) provides an approximation, and considering the expected impact of the neighboring second antiresonant mode on the torsional resonance, this value is considered a reasonable approximation for the modeling.

For the FEM modeling, the material constant matrices of $\mathbf{d}$ and $\mathbf{s}^E$ were taken from the same references as mentioned above, and the density used was $\rho = 4628$ kg/m³, based on literature data for congruent composition LN [9].

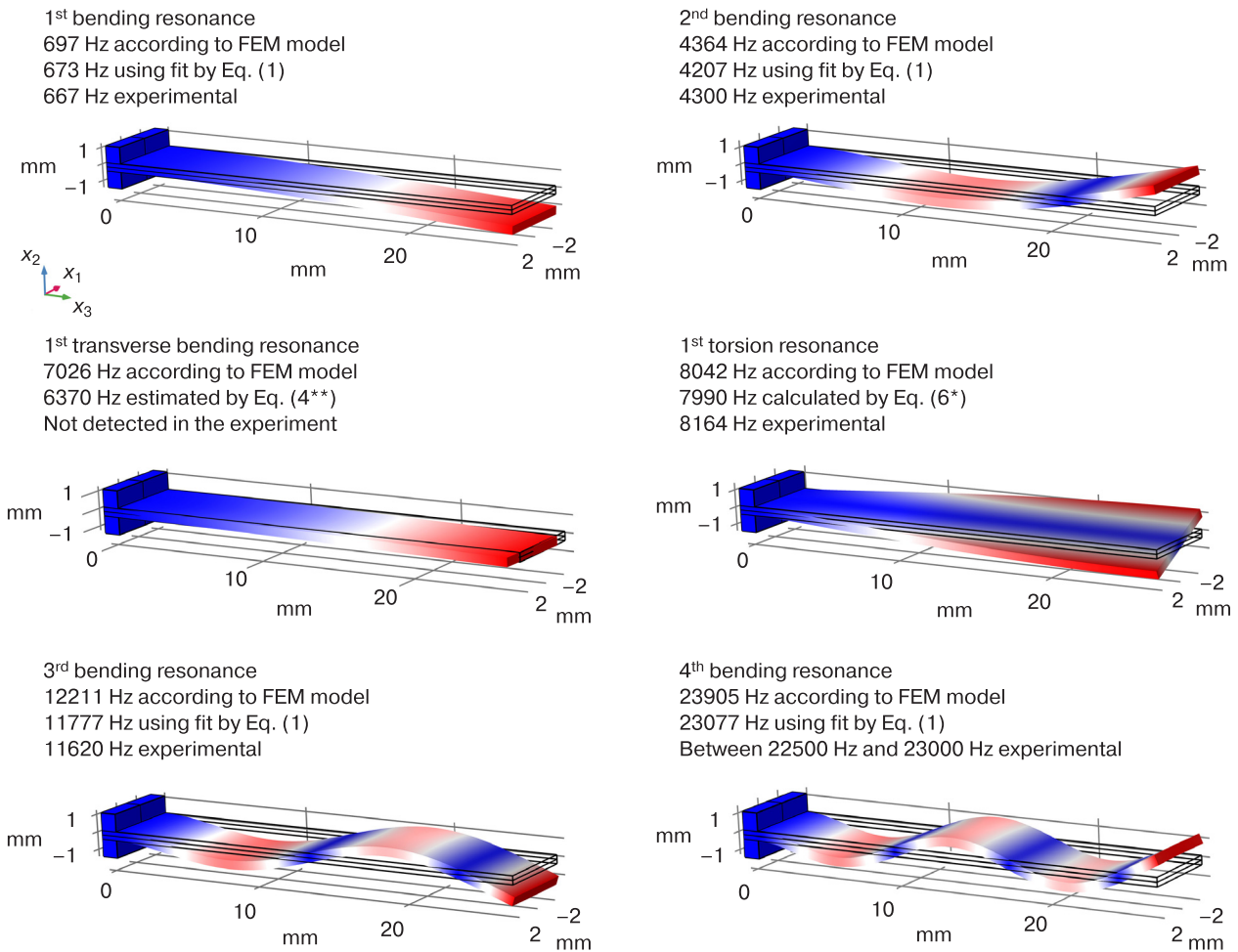


Figure 6. Shape representations of the deformed rectangular cantilever made from a BLN crystal, with the geometry described in the paper, at the first six resonant frequencies. The colorbar is arbitrary

The modeling was performed in the frequency range of 100 Hz to 25 kHz, and the eigenfrequencies of mechanical vibrations were determined. Within this range, four normal bending resonances, one lateral bending resonance, and one torsional resonance were identified. Representations of the crystal's deformation at each of these modes are shown in Fig. 6.

While FEM modeling using standard material parameters allows for predicting the positions of not only normal bending resonance frequencies but also lateral bending and torsional resonances, the challenge lies in accurately determining the frequency of the bending antiresonance. Moreover, the analytical 1D model provides more accurate predictions for the positions of bending resonances, particularly when non-linear least square optimization is applied to fit the parameters. However, the disadvantage of the analytical model is its limitation to 1D cases, meaning it is applicable only to narrow, thin, and long elastic beams.

Finally, we analyze the torsional vibrations of our BLN-based bimorph by examining the displacement measurements across the width near the crystal edge within the frequency range of 7.8 kHz to 8.8 kHz, as predicted by both FEM modeling and the analytical approach using

Eq. (6*). As discussed above, the actuator's design was selected such that this frequency range corresponds to the second bending antiresonance.

Despite the deviation of the actual value $l/w \approx 5.39$ from the required $l/w \approx 5.05$, torsional vibrations are observed as expected in the vicinity of the second bending antiresonance. This is clearly evident in Fig. 5b, represented by the green data points as a narrow peak at 8164 Hz. These points were measured at one of the corners near the free edge of the BLN-based bimorph. The leftward shift of the peak from the 2nd antiresonance is attributed to the aforementioned deviation in the l/w value. At the 2nd antiresonance, where the bent bimorph exhibits zero displacement at two points (the free end and $x_3 = 16.9$ mm), a slight feature related to the torsional resonance can also be observed in this dataset.

The displacement data across the frequency range and the investigated spatial positions are presented in Fig. 7. This data includes both the displacement magnitude and the displacement phase relative to the excitation signal.

Torsional displacements are readily observed in the frequency range of 8100 Hz to 8200 Hz, distinguishable due to the low normal-bending background caused by the proximity to the 2nd antiresonance. The displacement

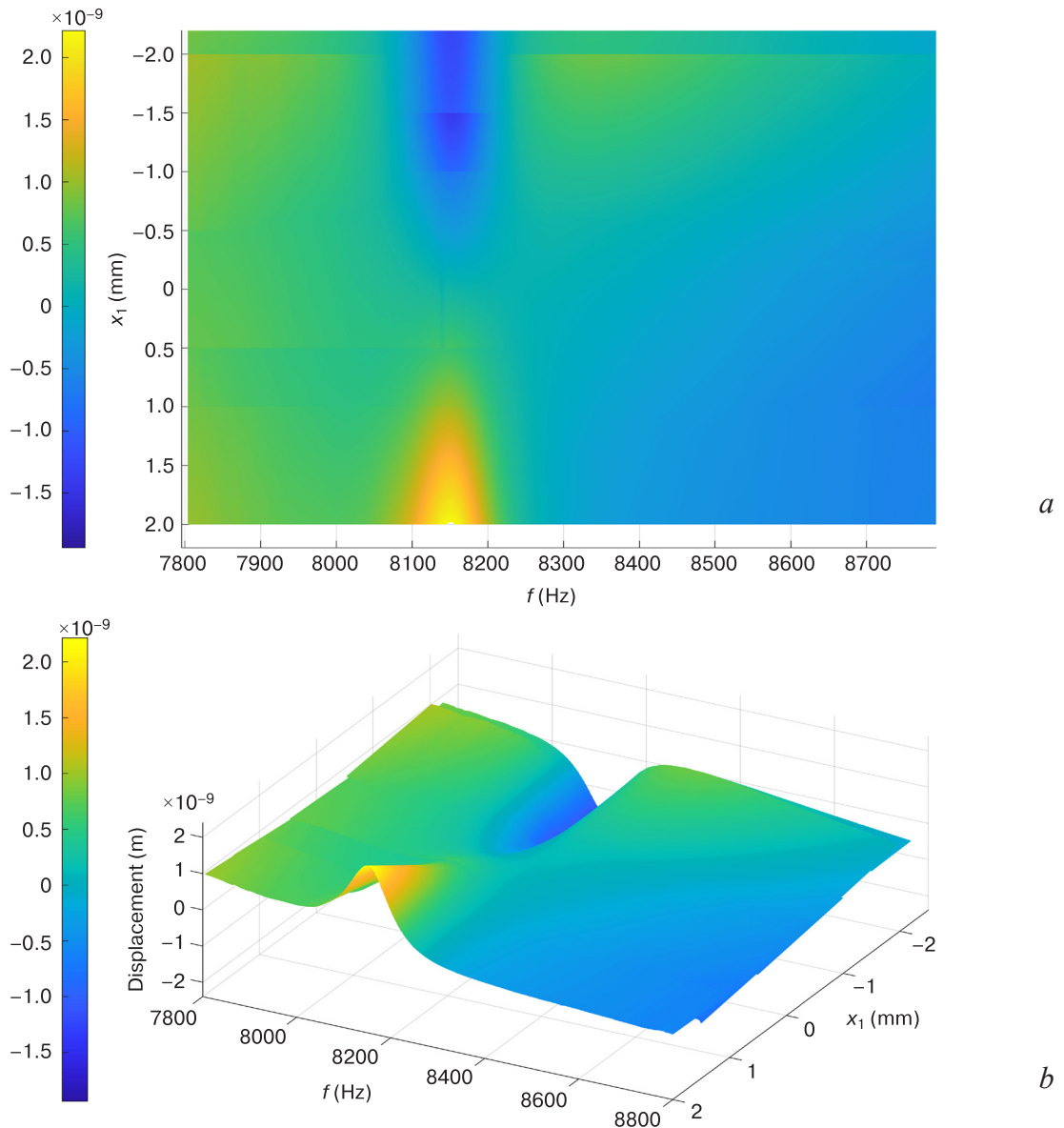


Figure 7. The representation of the displacement data across the frequency range of 7.8 kHz to 8.8 kHz and the spatial positions x_1 from -2.25 mm to 2.25 mm with $x_3 = 25.4$ mm for all the points

magnitude is maximized at the edges of the bimorph, reaching approximately 2.1 nm/V. From the “hillsides” of the green dataset in Fig. 5b, it is evident that torsional resonant vibrations – spontaneously excited by other displacements of the oscillating bimorph and piezoelectrically passive – can be observed near the bending antiresonance frequency only if the deviation of the actual l/w value from the required l/w ratio is not too large. For example, in our case, the displacement magnitude near the corner of the free end of the bimorph is approximately 2.1 nm/V. These displacements would be difficult to observe against the background of bending displacements that are 5 to 10 times larger, as seen at frequency shifts of approximately $\Delta f \approx \pm 2$ kHz from the 2nd antiresonance. However, these values are specific to the bimorph’s geometry.

A careful analysis of our results, particularly from Fig. 5a and Fig. 7, allows us to conclude that even relatively short BLN-based bimorphs (in our case $l/t \approx 51$) can be satisfactorily described by the 1D model, without the need for complex calculations to understand the displacements of the cantilevered bimorph actuator. For instance, in the case of the bimorph with the first bending resonance frequency of approximately 667 Hz, the actuator can operate within a range of 0 Hz to 200 Hz with an almost linear response. Additionally, each of the distinct resonant and antiresonant frequencies can be used for fast rotations of the free end at large angles, while avoiding interference with the torsional resonance of the cantilever beam, as the condition for observable interference between bending and torsion modes is established and easily testable.

4. Conclusions

In this study, we demonstrated the potential of BLN-based bimorph actuators as versatile and reliable systems for precision actuation. Through a combination of theoretical modeling and experimental validation, we established that the behavior of BLN-based bimorphs is effectively described by the 1D model proposed by Smits et al. [34, 36–38]. This accuracy arises from the absence of inter-grain or epoxy interfaces in these bimorphs, as well as the stable piezoelectric properties and ferroelectric domain structure of BLN crystals. The equations of this model enable not only the analytical calculation of displacement-frequency dependencies at various points along the bimorph's length but also the precise determination of bending resonance and antiresonance positions based on material constants. In certain cases, particularly in the low-frequency range, the analytical 1D model provides more accurate results than FEM modeling.

The strong correlation of BLN-based bimorphs with classical beam theory is further supported by their behavior at bending resonances and antiresonances, as well as at torsional resonances. This predictability offers significant practical advantages, particularly in enabling precise frequency tuning. Specifically, we demonstrated the feasibility of aligning the torsional resonance with the second bending antiresonance. This alignment, confirmed both theoretically and experimentally, highlights the potential for strategically tailoring frequency spectra for targeted applications, providing a considerable advantage in the development of advanced actuation systems and sensors. Notably, the torsional resonance, which is piezoelectrically passive, was first predicted and subsequently observed through displacement measurements across the

bimorph's width, validating the effectiveness of both analytical and FEM modeling approaches.

Our findings confirm that BLN crystals are an exceptional material for high-quality bimorph actuators, offering superior thermal stability and long-term reliability. Additionally, the availability of commercially produced LN crystals with globally consistent properties enhances their practicality for real-world applications.

The demonstrated capabilities of BLN-based bimorph actuators position them as strong candidates for advanced MEMS technologies [47]. This combination of simplicity and precision makes BLN-based bimorphs ideal for both theoretical and practical applications. Their ability to sustain high performance without requiring additional capacitive feedback loops opens new pathways for innovative sensing and actuation solutions across various fields. By leveraging the unique properties of BLN crystals, this work lays the foundation for the next generation of reliable, accurate, and thermally stable piezoelectric actuators and sensors.

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